Quantifying the Ripple:
Word-Of-Mouth and Advertising Effectiveness

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In this article the authors demonstrate how a customer lifetime value approach can provide a better assessment of advertising effectiveness that takes into account post purchase behaviors such as word-of-mouth. While for many advertisers word-of-mouth is viewed as an alternative to advertising, the authors show that it is possible to quantify the way in which word-of-mouth often complements and extends the effects of advertising. The authors provide a simple approach to the measurement of post-purchase word-of-mouth sales effects and demonstrate how firms may be underestimating advertising effectiveness by ignoring such effects. Their approach illustrates how customer lifetime value models can provide an important tool to assess the long-term effects of advertising campaigns.
Measuring Advertising Effectiveness: How Can We Improve?

Debate over how best to determine advertising effectiveness has continued unabated for decades as firms struggle to justify expenditures for one of the largest line items in the marketing budget. Participants in this debate often fall into two camps based on whether they focus on individual or aggregate level advertising response. Advocates for individual response models rightly argue that it is essential to understand how consumer’s process and react to an ad in order to better manage the development and deployment of advertising copy. Critics of individual level models argue that such models do not provide a clear linkage between the individual’s attitude formation and firm profits. In contrast, aggregate response models can be more clearly linked to profitability, but often provide few insights into the underlying processes driving sales growth and profits.

Models that bridge this gap between individual behavioral response models and aggregate models of long-term profitability have begun to emerge. The key to such models is an understanding of how an individual’s purchase behavior contributes to firm profits over time. In recent years, significant advances in the realm of customer profitability modeling have led to models that map individual customer behavior directly onto a measure of long-term firm value. These customer lifetime value (CLV) models enable firms to calculate the value of a customer acquired through advertising (or any other means) and to determine advertising’s effect on customer repurchase, willingness-to-pay, word-of-mouth, and other behaviors that drive profitability.  

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1 Customer lifetime value is calculated as the expected net profit that will be received from a customer over a specified time horizon (i.e., customer future revenue less the costs of acquiring and serving the customer), taking into account the customer’s retention rate, discounted to the present. For an overview of CLV models, see Berger and Nasr 1998, Jain and Singh 2002, and Rust, Lemon and Zeithaml 2004.
modeling is still in its infancy, a growing number of leading firms currently use the approach to guide resource allocation decisions for activities such as advertising (Brady 2000). As more firms turn to CLV modeling, it is useful to understand how these models can help advertisers improve measures of advertising effectiveness.

Current customer CLV models suggest two ways that advertising can improve long-term customer profitability. First, advertising can lower the cost of customer acquisition. More effective advertising leads to higher acquisition rates which, in turn, reduce the per person cost of acquisition. Second, advertising that focuses on brand building and repeat customers can increase add-on selling, the average price paid, and the customer retention rate. Reducing acquisition costs, increasing realized revenue per customer and improving retention rate will all increase the lifetime value of the customer.

CLV models offer an advantage over traditional aggregate sales response models because CLV models enable the firm to account for the multiple routes by which advertising affects firm profits. Moreover, such models explicitly incorporate individual customer behaviors (such as initial purchase intent, repurchase intent, and willingness-to-pay) into the profitability measure. By monitoring advertising’s effects on these behaviors, and the effects of these behaviors on customer and firm profitability, a more comprehensive assessment of advertising assessment can be achieved.

When CLV modeling is applied across the customer base to measure a firm’s customer equity (Blattberg and Deighton 1996; Rust, Zeithaml and Lemon 2000), it provides a powerful tool to provide a more complete picture of advertising effectiveness.2

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2 Customer equity is the total of the discounted lifetime values summed over all of the firm’s current and future customers (Blattberg and Deighton 1996, Rust, Lemon and Zeithaml 2004).
However, current models of CLV and advertising effectiveness may underestimate the extent of advertising’s effect on customer behavior. Such models (e.g., Rust, Lemon, Zeithaml 2004, Venkatesan and Kumar 2004) determine advertising effectiveness by examining the effect of marketing communications on individual customers. These models can undervalue advertising effectiveness because they assume that customers respond to advertising in isolation and do no interact with other customers. As we will demonstrate, this undervaluation (resulting from not taking customer interactions such as word-of-mouth into account) can be considerable and hence can significantly bias decisions regarding how much to spend on advertising campaigns.

**Word-of-Mouth: The Overlooked Advertising Ripple Effect**

There is broad agreement among managers, marketing researchers and sociologists that customer interactions through word-of-mouth (WOM) can have a major impact on consumer response to a product and the accompanying advertising (Arndt 1967; Danaher and Rust 1996; Herr, Kardes, and Kim 1991). For example, over 40% of American consumers actively seek the advice of family and friends when shopping for services such as doctors, lawyers, and auto mechanics (Walker 1995). WOM may play a more important role when the product in question is more risky or uncertain and when consumer's involvement with it is higher, and thus WOM has been found to be especially effective in driving the diffusion of new products (Rogers 1995) and in decision making regarding services (Murray 1991).

Like repeat purchases, the spread of WOM is largely driven by the customer's satisfaction with the product (Anderson 1998) and hence WOM (whether positive or
negative) can be seen as an integral part of the value the firm should associate with its customers. However, most of the empirical research to date has focused on analyzing the social processes related to WOM (see Buttle 1998 for a review) and not on quantifying the value it supplies the firm. In a recent exception, Hogan, Lemon and Libai (2003) have shown that early in the product lifecycle the value of the customer to the firm is often higher due to the WOM these customers supply, relative to the direct profits from their purchases.

Historically marketing managers have treated word-of-mouth as a “black box” whose workings are too complex to be able to articulate the link between this critical secondary purchase behavior and firm profits. This has led eminent researchers (Danaher and Rust 1996, Zeithaml 2000) to call for additional research into this link. The recent resurgence in the use of word-of-mouth campaigns, as evidenced by the growing use of referral reward programs, affiliate marketing and Internet based viral marketing campaigns emphasizes the need to better understand the value of this important social mechanism (see, for example, Biyalogorsky, Gerstner and Libai 2001, Gallaugher 1999, Libai, Biyalogorsky and Gerstner 2003, Kirby 2004).

Specifically, as marketing and advertising managers have known for many years, word-of-mouth often complements and extends the effects of advertising (Bayus 1985, Monahan 1984). Frequently it is the initial marketing communication that triggers a customer’s initial purchase. That purchase experience subsequently triggers the spread of word-of-mouth, as customers share their experience with others. The whole process would never be initiated without the customer’s initial exposure to the ad. Hence, it is important to account for the ripple effect created by this post-advertising word-of-mouth.
To do so, consider a customer who buys a product or service for the first time as a result of advertising. The lifetime value of this customer can be calculated (based upon projected retention rate, acquisition costs, frequency and value of each purchase, etc.). The question we seek to address is the following: how would the lifetime value of this customer (who first purchased as a result of advertising) change if we will take into account the subsequent word-of-mouth effects that follow the customer’s initial purchase?

In this study we demonstrate a simple approach for measuring the advertising ripple effect caused by WOM—i.e., the extent to which the effect of advertising is extended (or multiplied) by word-of-mouth communication. Using a customer lifetime value modeling approach, we show how the lifetime value of a customer acquired through advertising can increase several-fold when the effect of a customer’s WOM is included in the calculation. The approach, which can be implemented with a short survey, is straightforward and does not require extensive analytical training to implement. We demonstrate the approach using a sample of customers in the hair styling market to show the degree to which measures of advertising effectiveness may be underestimating the true impact of advertising by not accounting for the advertising ripple effect. We begin by showing how CLV modeling fits into the broader advertising effectiveness assessment.

Customer lifetime value is becoming a key metric for assessing the long-term value of the firm’s marketing efforts (Berger and Nasr 1998; Zahay et al. 2004). We contend that customer lifetime value is an appropriate metric for assessing advertising effectiveness because it recognizes the extent to which investments in advertising may
have effects in multiple (future) periods and are, therefore, recouped over time. This long-term perspective is more consistent with the notion of brand building in which the firm seeks to build an “asset” that it can leverage over many years (Ambler et al. 2002, Keller 2003). In addition, it provides a metric that is based on a net present value model for assessing future cash flows to the firm that can be linked more directly to shareholder value by aggregating individual CLV assessments across the entire customer base, i.e., the firm’s customer equity (Gupta, Lehmann and Stuart 2004, Hogan, Lemon and Rust 2002, Hogan et al. 2002). Describing advertising effects in terms of net present value helps marketing managers communicate the importance of advertising to the firm’s long-term success in terms that are most relevant to the CEO, CFO and CIO.

Quantifying the Advertising Ripple

In the model that follows, we incorporate word-of-mouth effects into a model of customer lifetime value to show how it is possible to quantify the advertising ripple effect. We also show how ignoring this ripple effect may underestimate the effectiveness of an advertising campaign. In order to quantify the advertising ripple effect it is necessary to map the social process by which word-of-mouth is transmitted through the customer base onto a customer lifetime value (customer profitability) framework. To accomplish this we introduce the concept of customer strings.

A customer string is defined as: a set of purchases by customer(s) linked by the word-of-mouth communication processes initiated by the focal customer of the CLV calculation.

The primary customer string includes all of the purchases of the focal customer whose lifetime value is being assessed. The secondary customer string would include all of the
purchases by customers that were acquired as a result of positive word-of-mouth spread by the customer in the primary string. It is possible, therefore, that the secondary string could contain the purchases of more than one customer because the primary customer may affect more than a single individual through his/her word-of-mouth. Tertiary customer strings would include all of the customers that were acquired as a result of WOM spread by customers in the secondary string, and so on (see Figure 1).

Table 1 provides a numerical estimate, for six consecutive periods, of the number of customers purchasing each period (following the initial ad-based purchase by the focal customer) broken down by customer string. To better illustrate the effect of WOM communications on customer acquisition, we have identified the newly acquired customers in each period separately. We impose two restrictions. Given that an individual must receive WOM before his or her purchase can be influenced by WOM, we impose the restriction that individuals affected by WOM in one period start to buy in the next period. Second, we restrict the spread of WOM to those who have actually purchased the product. These restrictions can be relaxed in future applications.  

Consider the first row of Table 1, which contains the primary string (i.e., the total purchases made by the originating, focal customer). Not all new customers will be retained by a firm so, in accordance with convention, we have adjusted the “number” of customers in each subsequent period by the retention rate, \( r \) (Reichheld 1996). We

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For example, the first restriction suggests that if a customer first purchases in period 2, they will not spread WOM until period 3. The second restriction does not account for those individuals who hear about the product, tell others, but do not purchase themselves. Together, these restrictions provide a more conservative estimate of the effect of WOM on customer profitability.
therefore model the number of customers in the first period as 1.0 (or 100%), the number in the second period as \( r \) (the percent of customers retained each period), the number in the third period as \( r \cdot r = r^2 \), and so forth. This basic valuation process applies to each subsequent customer string.\(^4\)

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**Insert Table 1 Here**

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Now consider the secondary string that contains customers acquired as a result of the word-of-mouth spread by the primary string customer (see Table 1, second row). In the second period, new customers are acquired from the WOM spread by the original customer. We denote the number of newly acquired customers as \( w \). Under our approach \( w \) represents the average number of customers per period that an existing customer converts to buyers through WOM. Being conservative, we consider only customers who otherwise (i.e., without WOM) would not have purchased. Note that the value of \( w \) will typically be small, as only some consumers will spread positive WOM (and those customers will not spread WOM every period), and as only a portion of WOM communications will convert potential buyers into actual buyers.

The number of these customers declines with time according to the same retention process that applied to the primary string. In the third period, the number of new customers is determined by the \( r \) customers remaining in the primary string. Thus, the number of new buyers is \( wr \) and the total number of customers is \( 2wr \). In the fourth period, the number of new customers is determined by the \( r^2 \) buyers of the primary string.

\(^4\) For example, if the retention rate is 90%, then the number of customers in period two would be \( r = .9 \) (or 90%), and in period three would be \( r^2 = .9 \times .9 = .81 \) (or 81%).
and thus the number of new customers is \( wr^2 \). These customers are then added to the \( 2wr^2 \) left of the \( 2wr \) customers already purchasing in the second string for a total of \( 3wr^2 \) customers. The same logic used to determine the number of customers in the secondary string can be used to complete the table for as many customers as desired.

In Table 2 we show how to convert the number of customers acquired through the ripple effect to monetary value assuming each purchase yields an average of \( x \) dollars and that the discount rate is \( d \). Having converted customers acquired to dollar value we can determine the advertising ripple effect (the word-of-mouth multiplier) by dividing the values of all of the higher strings into the value of the first row representing the value of the initial customer acquired through advertising.

Table 2 can be modified to reflect a relaxing of the assumptions. For example, parameters may be allowed to change over time and other parameters may be added. One of the advantages of the simple approach, however, is that it allows us to arrive at a straightforward expression for the long-run WOM multiplier. One might assume that if the process continued indefinitely, then the WOM multiplier would be extremely high. In fact, as we show in the Appendix, in many cases in the long run the whole process described in Table 2 converges to a simple expression of the WOM multiplier.

The simple expression of the word-of-mouth multiplier that stems from the process in Table 2 is given by:

\[
(1) \quad \text{WOM multiplier} = \frac{1 + d - r}{1 + d - w - r}
\]
Equation (1) provides advertisers with a “back of the envelope” assessment of the magnitude by which WOM affects the economic value of an advertising induced purchase. To get a feel for the magnitude of the results, consider a case in which the yearly retention rate is 0.8 (reported to be the average retention rate across many U.S. industries, see Reichheld 1996) and the yearly discount rate is 0.1. Figure 2 presents the relationship between different values of \( w \), the yearly WOM conversion rate, and the WOM multiplier. We see that when \( w \) is 0 the multiplier is 1 since there is no WOM to generate the ripple effect. However, when the yearly conversion rate \( (w) \) is 0.2, for example, the WOM multiplier \( M \), is: \[ M = \frac{1 + 0.1 - 0.8}{1 + 0.1 - 0.8 - 0.2} = 3 \]. Thus, under this combination of parameters, the real value that an ad generates is \textit{three} times more than the value if WOM is not taken into account.

\[ \text{Insert Figure 2 Here} \]

\textbf{An Empirical Illustration of the Ripple}

The approach described above can be easily implemented by many product and service firms. To provide an example of such an implementation, we now describe how this approach has been used to quantify the advertising ripple effect for the hairstyling market in a U.S. city using a cross-sectional survey to estimate the relevant model parameters. We selected hairstyling services as the market context because prior research has found that word-of-mouth plays an important role in the consumer’s purchase decision for personal services such as hairstyling (Walker 1995).
Data Collection and Sample

The data were collected from a convenience sample of students attending a private university in the eastern United States. Although student subjects are often considered unsuitable for advertising research, this is not an issue in this research because our objective is to illustrate the method rather than to generalize the results to a broader population. Students were approached on campus by the researcher, and asked if they would be willing to participate in a survey. Students who agreed to participate were entered into a drawing for one of eight $50 gift certificates to the student bookstore. Approximately 90% of the students approached agreed to complete the survey, suggesting a low potential for non-response bias. Respondents included a mix of undergraduate and graduate students who were screened based on whether they had their hair styled locally or at their permanent home. A total of 204 surveys were administered of which 7 were disqualified for incomplete answers resulting in 197 completed surveys. Each respondent was questioned about the name of the establishment where he/she had their hair styled most recently. The completed surveys were then separated into groups based on this designation. The respondents mentioned a total of 49 different establishments, although the six most frequented establishments accounted for 122 of the 197 completed surveys. These six establishments were then segregated for subsequent analysis.

The most frequented establishment, which we refer to as “Good Cuts”, was mentioned by 61 of the 122 respondents with the other 61 surveys being divided amongst the other five shops, which we collectively describe as “Premium Competitors.” The characteristics of the salons and the clientele differed significantly between the two
groups. For example, the average price paid for a hair styling session at Good Cuts was $13.81 compared to $22.05 at the Premium Competitors (p<.01). Premium Competitors customers were more satisfied with their most recent purchase experience with a satisfaction score of 1.15 for Premium Competitors versus 1.27 for Good Cuts (with 1 being extremely satisfied and 5 being extremely dissatisfied, p < .05). Consumers at Good Cuts spent less time each day styling their hair compared to the competitors’ customers (6.7 minutes/day versus 12.09 minutes/day, p < .01).

Estimating Model Parameters

Our approach to estimating w (the per-period number of new customers acquired via word-of-mouth) combines data from new and repeat customers to capture the conversion rate attributable to word-of-mouth. Repeat customers were asked about the number of positive WOM communications they had engaged in following their last purchase. Then new customers were asked about the importance of WOM communications in their initial purchase decision. This information from new and repeat customers was then combined to yield a measure of w.

Each respondent was asked to rank the importance of various factors including word-of-mouth, advertising, and coupons in their purchase decision. Only those customers that listed word-of-mouth as the sole factor in their decision were classified as a conversion. This approach does not include respondents for which WOM was important but not the sole influence on the purchase decision and thus, it tends to understate the impact of word-of-mouth. The methodology can accommodate less conservative approaches enabling managers to use their judgment about what is

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5 Informal research of local hair salons supported these prices for a hair cut, and supported the idea that a significant portion of the salons’ clientele is from the large (10,000 student) university community.
appropriate for their firm and specific market conditions. Using a detailed questionnaire we could also relax some of the assumptions of the general model. For example, we could make our estimation more precise by estimating how the monthly retention rate $r$ and WOM conversion effect $w$ change with time, and incorporated that into a table based on the general structure of Table 2.

Based on these results, we calculated the three-year lifetime value for customers in both samples. We opted for three years because most students leave the area upon graduation and because only a small percentage of the customer base was active in the third year (13% for both Good Cuts and Premium Competitors sample). Moreover, three years is within the recommended lifetime for general CLV applications (Venkatesan and Kumar 2004). The lifetime value of the direct purchases of a Good Cuts customer (not including WOM) is $82.26 using a discount rate of 15% per annum. The value of WOM for these customers was $80.15, suggesting a total average customer lifetime value of $162.41. The WOM accounts for 49.4% of the total CLV; hence the WOM multiplier for Good Cuts is about 2.

In comparison, the value of direct purchases of the Premium Cuts customer is $139.01, reflecting the higher prices paid relative to Good Cuts. The value of WOM for Premium Competitors was $298.21, for a total CLV of $437.22. The WOM accounts for 68.2% of the total lifetime value of the customer in this case, leading to a WOM multiplier of 3.2 for Premium Competitors. Thus, the advertising effectiveness of the premium competitors will be approximately 57% higher than that of Good Cuts because of the difference in the advertising ripple effect.
To understand the importance of considering this ripple effect, consider the following examples, based upon the above example. If Good Cuts advertises and acquires 100 new customers as a result of its advertising efforts, each with an average (direct) CLV of $82.26, direct approaches to measuring advertising efforts (in terms of purchase behaviors) would suggest that the advertising produced a return of $8226. However, if one considers the additional value of the WOM that these new customers will spread to others ($80.15 on average), the value of the advertising campaign is actually about twice that, or $16,421. The effect is even more striking for the premium competitors. Using a similar scenario, if these firms acquired 100 new customers, the total CLV for these customers when just accounting for the direct effects would be $13,901. However, when the effects of the word-of-mouth that these new customers will spread is also considered, the effect of advertising is much greater: $43,722.

Discussion, Limitations, and Conclusion

The advertising ripple effect represents an unexplored component of the factors driving the effectiveness of advertising campaign. As we have demonstrated here, the word-of-mouth generated following an advertising-based purchase can represent a considerable portion of the economic worth of a promotion. Not taking this additional value into account can strongly bias any analysis of advertising effectiveness and the consequent managerial implications that follow such an examination.

The approach we presented above is clearly not the last word on this subject. In this article we have followed simple assumptions, consistent with the basic models of the customer lifetime value literature. This allowed us to derive a simple expression for the
word-of-mouth multiplier. In practical situations, as in our empirical illustration, following the general approach suggested here, such assumptions could be relaxed to yield a more exact analysis for a specific situation. In addition, other factors can be taken into account. For example, we examined the effect of positive WOM and not that of negative WOM. The analysis and measurement of products generating negative WOM may be somehow more complicated than the case with positive one, but nevertheless it is an important avenue for future exploration. In addition, future research should examine the extent to which customers acquired through advertising differ in their propensity to spread word-of-mouth from customers acquired via other means.

It is important to consider the industries and contexts in which the advertising ripple effect can be quantified. We have described an example in a consumer services industry, in which customers frequently repurchase the service. Although we have not collected data in other industries, we believe that the model will extend to many consumer services and consumer products that are repurchased on a somewhat regular basis (as long as word-of-mouth plays a role in the decision making of some customers), and should extend to regularly purchased products and services in business-to-business industries as well. Recent research by Dye (2000) suggests that approximately two-thirds of the U.S. economy is at least partially affected by word-of-mouth. However, for infrequently purchased products and services, it is more difficult to ascertain the retention rate for individual customers and the model would, therefore, be estimated with more uncertainty. Additional research is needed to understand the advertising ripple effect in such environments.

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6 See Wangenheim and Bayón 2003 for an example of quantifying the effect of a customer word-of-mouth referral in both business-to-business and business-to-consumer markets in the energy industry in Germany.
A ripple effect analysis is not limited to the approach described here. An encouraging development in this regard is that of complex systems methods, which are very suitable for WOM analysis, and are increasingly making their way into the social science literature. Originally utilized in life sciences such as Physics and Biology, methods such as cellular automata, in which a system is analyzed through a simulated “would-be world” that allows testing of a wide range of scenarios, can be used for marketing applications. Such analysis enables researchers to understand how local interactions between individuals aggregate to market level phenomena. In depth analysis of the advertising ripple effect is a promising application of such tools (Goldenberg, Libai and Muller 2001).

The bottom line is that the advertising ripple effect can be quantified. As marketers in general consistently aim to make WOM approaches and analysis part of their marketing strategies, advertising researchers should also make WOM analysis and its economic implications an important research issue.
Figure 1: Mapping the WOM Communication Process

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>m-1 ... m</th>
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</thead>
<tbody>
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<td>Primary String</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Secondary String</td>
<td></td>
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<tr>
<td>Tertiary String</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Nth level Strings</td>
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</tbody>
</table>

Conventional CLV + Value of WOM Strings = Total CLV
Figure 2: The Effect of WOM Conversion Rate on the Ripple Effect
Table 1: Number of Customers Acquired in the Ripple Effect

<table>
<thead>
<tr>
<th>String</th>
<th>Per. 1</th>
<th>Per. 2</th>
<th>Per. 3</th>
<th>Per. 4</th>
<th>Per. 5</th>
<th>Per. 6</th>
</tr>
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<td>1</td>
<td>r</td>
<td>r²</td>
<td>r³</td>
<td>r⁴</td>
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<tr>
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<td>New</td>
<td>w</td>
<td>wr</td>
<td>w²r</td>
<td>w³r²</td>
<td>w⁴r³</td>
</tr>
<tr>
<td></td>
<td>Total</td>
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<td>2wr</td>
<td>3wr²</td>
<td>4wr³</td>
<td>5wr⁴</td>
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<tr>
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<td>w²</td>
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<td>3w²r²</td>
<td>4w²r³</td>
<td>10w²r³</td>
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<tr>
<td></td>
<td>Total</td>
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<td>6w²r²</td>
<td>10w²r³</td>
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<tr>
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<td>New</td>
<td>w³</td>
<td>3w³r</td>
<td>6w³r²</td>
<td>10w³r³</td>
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<tr>
<td></td>
<td>Total</td>
<td>w³</td>
<td>4w³r</td>
<td>10w³r²</td>
<td></td>
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<tr>
<td>5</td>
<td>New</td>
<td>w⁴</td>
<td>4w⁴r</td>
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<td>Total</td>
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<tr>
<td>6</td>
<td>New</td>
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<td></td>
<td>Total</td>
<td>w⁵</td>
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</table>
Table 2: Present Value of the Advertising Ripple Effect

<table>
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<tr>
<th>String</th>
<th>Per. 1</th>
<th>Per. 2</th>
<th>Per. 3</th>
<th>Per. 4</th>
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<td>$x \cdot \frac{r}{1 + d}$</td>
<td>$x \cdot \left( \frac{r}{1 + d} \right)^{2}$</td>
<td>$x \cdot \left( \frac{r}{1 + d} \right)^{3}$</td>
<td>$x \cdot \left( \frac{r}{1 + d} \right)^{4}$</td>
<td>$x \cdot \left( \frac{r}{1 + d} \right)^{5}$</td>
</tr>
<tr>
<td>2</td>
<td>$x \cdot \frac{w}{1 + d}$</td>
<td>$x \cdot \frac{2wr}{(1 + d)^2}$</td>
<td>$x \cdot \frac{3wr^2}{(1 + d)^3}$</td>
<td>$x \cdot \frac{4wr^3}{(1 + d)^4}$</td>
<td>$x \cdot \frac{5wr^4}{(1 + d)^5}$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>3</td>
<td>$\ldots$</td>
<td>$x \cdot \frac{w}{(1 + d)} \left[ \frac{w}{1 + d} \right] \left[ \frac{w}{1 + d} \right] \left[ \frac{w}{1 + d} \right] \left[ \frac{w}{1 + d} \right] \left[ \frac{w}{1 + d} \right]$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
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<td>6</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$x \cdot \frac{w}{(1 + d)}$</td>
</tr>
</tbody>
</table>
Appendix:

How the ripple process converges

To arrive at a long run WOM multiplier, consider the sum of the values of the columns (periods) in Table 2. It is easy to see that the columns sum to the following time series:

\[ \left( \begin{array}{c}
1 + \frac{w + r}{1 + d} + \left( \frac{w + r}{1 + d} \right)^{1} + \left( \frac{w + r}{1 + d} \right)^{2} + \left( \frac{w + r}{1 + d} \right)^{3} + \ldots \\
\sum_{i=0}^{\infty} \left( \frac{w + r}{1 + d} \right)^{i}
\end{array} \right) \]

if \( \frac{w + r}{1 + d} < 1 \) this geometric series converges in the long run to

\[ \frac{1}{1 + d - w - r} \]

Given that \( w \) represents average WOM conversions per period across the whole customer base, not only the highly satisfied ones, we expected it to be much smaller than 1 for most cases, \( \frac{w + r}{1 + d} < 1 \) is expected to be true in most cases.

Similarly, the first row (primary string) converges to

\[ \frac{1}{1 + d - r} \]

Hence the WOM multiplier is

\[ \frac{\frac{1}{1 + d - r}}{\frac{1}{1 + d - w - r}} = \frac{(A2)}{(A3)} = \frac{1 + d - r}{1 + d - w - r} \]
References


